## Cambridge IGCSE ${ }^{\text {TM }}$



## CAMBRIDGE INTERNATIONAL MATHEMATICS

Paper 4 (Extended)
October/November 2020
2 hours 15 minutes
You must answer on the question paper.
You will need: Geometrical instruments

## INSTRUCTIONS

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do not use an erasable pen or correction fluid.
- Do not write on any bar codes.
- You should use a graphic display calculator where appropriate.
- You may use tracing paper.
- You must show all necessary working clearly and you will be given marks for correct methods, including sketches, even if your answer is incorrect.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- For $\pi$, use your calculator value.


## INFORMATION

- The total mark for this paper is 120 .
- The number of marks for each question or part question is shown in brackets [ ].

This document has 20 pages. Blank pages are indicated.

## Formula List

For the equation $\quad a x^{2}+b x+c=0 \quad x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}$

Curved surface area, $A$, of cylinder of radius $r$, height $h$.
$A=2 \pi r h$

Curved surface area, $A$, of cone of radius $r$, sloping edge $l$.
$A=\pi r l$

Curved surface area, $A$, of sphere of radius $r$.

Volume, $V$, of pyramid, base area $A$, height $h$.

Volume, $V$, of cylinder of radius $r$, height $h$.

Volume, $V$, of cone of radius $r$, height $h$.
$V=\frac{1}{3} \pi r^{2} h$

Volume, $V$, of sphere of radius $r$.
$V=\frac{4}{3} \pi r^{3}$


$$
\begin{aligned}
& \frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C} \\
& a^{2}=b^{2}+c^{2}-2 b c \cos A \\
& \text { Area }=\frac{1}{2} b c \sin A
\end{aligned}
$$

Answer all the questions.
1 Adam and Brenda share $\$ 560$ in the ratio Adam : Brenda $=4: 3$.
(a) Show that Adam receives $\$ 320$.
(b) Adam spends $15 \%$ of his $\$ 320$ on some software.

Calculate how much Adam spends on this software.

> \$
(c) In a sale, Brenda buys a computer for $\$ 179.40$.

This is $8 \%$ less than the original price.

Calculate the original price of the computer.
\$
(d) Adam spends a further $\$ 29.60$ on a train ticket.

Adam and Brenda then work out how much money each of them has left.
Show that Adam has 4 times as much left as Brenda.

2 (a) Find the size of one interior angle of a regular polygon with 45 sides.
(b)


NOT TO
SCALE

In the diagram, $A, B, C$ and $D$ lie on the circle.
$T A$ is a tangent to the circle at $A$.
Angle $T A D=75^{\circ}$ and angle $D A C=35^{\circ}$.
Find
(i) angle $A C D$,

Angle $A C D=$
(ii) angle $A B C$.
(c)


NOT TO SCALE

In the diagram, $D E$ is parallel to $B C$.
(i) Complete the statement.

Triangle $A D E$ is $\qquad$ to triangle $A B C$.
(ii) $A E=6 \mathrm{~cm}, E C=3 \mathrm{~cm}$ and $D B=2 \mathrm{~cm}$.

Calculate the length of $A D$.

$$
A D=
$$

$\qquad$ cm [3]
(iii) The area of triangle $A D E$ is $9 \mathrm{~cm}^{2}$.

Calculate the area of triangle $A B C$.

3 (a) Eva records her science homework marks during the school year.
The table shows the results.

| Homework mark | 5 | 6 | 7 | 8 | 9 | 10 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency | 2 | 7 | 11 | 13 | 5 | 2 |

Find
(i) the range,
(ii) the mode,
(iii) the median,
$\qquad$
(iv) the lower quartile,
(v) the mean.
(b) Frank compares the science marks, $x$, with the mathematics marks, $y$, of ten students. The table shows the results.

| Science mark $(x)$ | 14 | 18 | 15 | 20 | 17 | 17 | 18 | 15 | 18 | 15 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Mathematics mark $(y)$ | 13 | 19 | 16 | 19 | 17 | 14 | 17 | 15 | 15 | 12 |

(i) Complete the scatter diagram.

The first six points have been plotted for you.

(ii) What type of correlation is shown on the scatter diagram?
(iii) Find the equation of the line of regression, giving $y$ in terms of $x$.

$$
\begin{equation*}
y= \tag{2}
\end{equation*}
$$

(iv) Another student's science mark is 16 .

Use your answer to part (b)(iii) to find an expected mathematics mark for this student.
(c) Georgio records the time, $t$ minutes, he takes to complete each of 40 pieces of mathematics homework.
The table shows his results.

| Time $(t$ minutes $)$ | $0<t \leqslant 10$ | $10<t \leqslant 15$ | $15<t \leqslant 20$ | $20<t \leqslant 40$ |
| :--- | :---: | :---: | :---: | :---: |
| Frequency | 9 | 20 | 6 | 5 |

Calculate an estimate of the mean.

4 The Venn diagram shows the number of students who like sweets $(S)$ and the number of students who like nuts ( $N$ ).

(a) (i) Find the number of students who like nuts.
(ii) Find the number of students who like sweets or nuts but not both.
$\qquad$
(b) (i) Find $\mathrm{n}(\mathrm{U})$.
(ii) Find $\mathrm{n}(S \cup N)$.
$\qquad$
(iii) Find $\mathrm{n}\left(S^{\prime} \cup N\right)$.
$\qquad$
(c) One of these students is chosen at random.

Find the probability that this student likes nuts but not sweets.
$\qquad$
(d) Two of these students are chosen at random.

Find the probability that they both like sweets and nuts.
(e) Two students who like sweets are chosen at random.

Find the probability that they both also like nuts.

5 (a) Carla invests $\$ 600$ at a rate of $1.8 \%$ per year compound interest.
Calculate the value of Carla's investment at the end of 7 years.

> \$
(b) Dominic wants to invest his money so that it will double its value in 17 years.

Find the lowest possible rate of compound interest per year that will give Dominic this result. Give your answer correct to 1 decimal place.
(c) Each year, the population of a village is decreasing at a rate of $4 \%$ of its value at the beginning of that year.
The population is now 2120 .
Find the number of complete years since the population was last greater than 2700 .


NOT TO
SCALE

The diagram shows a solid made from a cylinder and two hemispheres. The cylinder has radius 3 cm and length 11 cm .
Each hemisphere has radius 3 cm .
(a) Find the volume of the solid.

Give your answer in terms of $\pi$.
$\qquad$ $\mathrm{cm}^{3}$ [3]
(b) The solid is melted down and all the metal is used to make a cylinder of length 15 cm .
(i) Use your answer to part (a) to find the radius of this cylinder.
(ii) A rectangular tank contains water.

The base of the tank measures 20 cm by 10 cm .
The cylinder is placed in the tank so that it is completely covered by water. No water overflows the tank.

Calculate the increase in the depth of the water in the tank.

7 (a) (i) Factorise $a^{2}-b^{2}$.
(ii)


NOT TO
SCALE

The diagram shows two squares.
The difference between the areas of the squares is $7.41 \mathrm{~cm}^{2}$.
The difference between the lengths of the sides of the squares is 1.3 cm .
Find the area of the larger square.
$\mathrm{cm}^{2}$
(b) (i) Factorise $x^{2}-5 x-24$.
(ii)


NOT TO
SCALE

The area of rectangle $A$ is $15 \mathrm{~cm}^{2}$ greater than the area of rectangle $B$.
Find the area of rectangle $A$.

8 (a)

(i) On the diagram, sketch the graph of $y=1.5^{-x}$ for $-2 \leqslant x \leqslant 2$.
(ii) Solve the inequality $0.5 \leqslant 1.5^{-x} \leqslant 1$.
(iii) Solve the equation $1.5^{-x}=x^{2}$ for $-2 \leqslant x \leqslant 2$.
$\qquad$
(iv) On your diagram shade the regions where $1.5^{-x}<x^{2}$ for $-2 \leqslant x \leqslant 2$.
(b)


The diagram shows a sketch of the graph of $y=\frac{-2 x+b}{x+a}$.
The asymptotes of the graph are $x=2$ and $y=-2$.
The graph passes through the point $(0,2)$.
Find the value of $a$ and the value of $b$.

$$
\begin{align*}
& a= \\
& b=
\end{align*}
$$

$\qquad$
(c)

$\mathrm{f}(x)$ is a function such that

- the asymptotes of the graph are $x=a, x=b$ and $y=k$
- when $x<a$, the gradient of the graph is positive
- when $x>b$, the gradient of the graph is negative
- $M$ is the only local maximum point
- the graph does not cross any asymptote.

On the diagram sketch the graph of $y=\mathrm{f}(x)$.
$9 \quad \mathbf{p}=\binom{-1}{3} \quad \mathbf{q}=\binom{2}{-1}$
$A$ is the point $(3,4)$.
(a) Find $\mathbf{p}-\mathbf{q}$.
(b) $A$ is translated onto $H$ by the vector $\mathbf{p}$.

Find the coordinates of $H$.
$\qquad$
(c) $J$ is translated onto $A$ by the vector $\mathbf{q}$.

Find the coordinates of $J$.
$\qquad$
(d) Find the coordinates of the mid-point of $H J$.
$\qquad$
(e) Find the length of $H J$.

$$
\begin{equation*}
H J= \tag{3}
\end{equation*}
$$

(f) A line $L$, parallel to the vector $\mathbf{q}$, has gradient $-\frac{1}{2}$.

Find the equation of the line perpendicular to the line $L$ that passes through the point $A$.

10 (a)

(i) Describe fully the single transformation that maps shape $T$ onto shape $V$.
$\qquad$
$\qquad$
(ii) Reflect shape $T$ in the line $y=-2$.
(iii) Stretch shape $T$ by a factor of 2 with the $x$-axis invariant.
(b)
$\mathrm{f}(x)=\log (x) \quad \mathrm{g}(x)=\log \left(x^{3}\right)$
Describe fully the single transformation that maps the graph of $y=\mathrm{f}(x)$ onto the graph of $y=\mathrm{g}(x)$.
$\qquad$
$\qquad$

## Question 11 is printed on the next page.

$$
\mathrm{f}(x)=x^{3} \quad \mathrm{~g}(x)=3^{x}
$$

(a) Find $\mathrm{g}(2)-\mathrm{f}(2)$.
(b) Find $x$ when $\mathrm{g}(x)=\frac{1}{9}$.
(c) Write $x-\frac{1}{\mathrm{f}(x)}$ in terms of $x$.

Give your answer as a single fraction.
(d) Find $\mathrm{f}^{-1}(x)$.

$$
\mathrm{f}^{-1}(x)=
$$

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